# A Comparison Study of Spin Transfer Torque and Spin-Orbit Torque Based Stochastic Computing Using Computational Random Access Memory (SC-CRAM) 

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#### Abstract

Stochastic computing (SC) is a probabilistic computing method that benefits from high noise resiliency and can perform complex arithmetic functions with a small number of logic gates, thus making it a promising solution for next generation neuromorphic systems. However, generating random bit-streams using CMOS based technologies requires a large number of transistors, thus drastically increasing the total computation delay and energy consumption. In our previous work, we presented a solution where stochastic computation and stochastic bit generation were embedded within the same memory cell using a spintronic based in-memory computing architecture called computational random access memory, which we called SC-CRAM. In this work, we evaluate and compare the performance of SC-CRAM between spin transfer torque (STT) and spin orbit torque (SOT) switching mechanisms using key parameters of magnetic tunnel junctions from the research labs, the current industry standards, and the project performance metrics. Our calculations showed that, based on current trends, the performance of SC-CRAM can be further optimized by utilizing the SOT switching mechanism when the tunneling magnetoresistance ratio of the magnetic tunnel junction pillars increases and the resistance-area product of the pillars is minimized. SC-CRAM benefits from high noise resilience and small array sizes and our results demonstrate that the its performance metrics can be enhanced.


Index Terms-In-memory computing, magnetic tunnel junctions, spin orbit torque, spin transfer torque, stochastic computing.

## I. Introduction

HARDWARE REALIZATION of biologically plausible neuromorphic computing algorithms is very difficult, and in some cases impossible, on modern computing platforms. This is due to the large number of transistors required to perform neuronal and synaptic tasks as well as the large latencies and energy dissipation caused by the Von-Neumann bottleneck [1]. There are certain neuromorphic tasks, such as recognition, classification, and prediction, that do not require high mathematical precision, but rather, require extracting key information and approximations from large data sets. Therefore, recent studies have examined beyond CMOS technologies and information encoding schemes as alternatives to the current CMOS based computing platforms for various neuromorphic applications [2-4]. In particular, various probabilistic computing schemes have been attractive solutions for these types of tasks since these methods are highly resilient to noise and small variations in the input data [5]. Spintronics is a branch of beyond CMOS technologies [6-7] that are promising solutions for probabilistic computing schemes and have been proposed as random number generators [8-11].

One particular probabilistic computing scheme that has shown to be promising for neuromorphic computing applications is stochastic computing (SC) [12]. In SC, each realvalued number $\mathrm{x}(0 \leq \mathrm{x} \leq 1)$ is represented by a sequence of random bits, each of which has probability x of being one and probability $1-\mathrm{x}$ of being zero. These bits can either be serial streaming on a single wire or in parallel on a bundle of wires. When serially streaming, the signals are probabilistic in time; when in parallel, they are probabilistic in space. Consider the operation of multiplication implemented in SC. It consists of but a single AND gate. The inputs are two independent input
stochastic bit streams A and B. The number represented by the output stochastic bit stream C is

$$
\begin{gather*}
c=P(C=1)=P(A=1 \text { and } B=1)  \tag{1}\\
=P(A=1) P(B=1)=a \cdot b
\end{gather*}
$$

The probability of getting a one at the output, $\mathrm{P}(\mathrm{C}=1)$, is equal to the probability of simultaneously getting ones at the inputs, namely, $\mathrm{P}(\mathrm{A}=1)$ times $\mathrm{P}(\mathrm{B}=1)$. So the AND gate multiplies the two values represented by the stochastic bit streams. Consider the operation of addition implemented in SC. It is not feasible to add two probability values directly; this could result in a value greater than one, which cannot be represented as a probability value. However, we can perform scaled addition with a multiplexer (MUX), a digital construct that selects one of its two input values to be the output value, based on a third "select" input value. Call the select input S and the two data inputs A and B . When $\mathrm{S}=1$, the output $\mathrm{C}=\mathrm{A}$. Otherwise, when $\mathrm{S}=0$, the output $\mathrm{C}=\mathrm{B}$. With the assumption that the three input stochastic bit streams $\mathrm{A}, \mathrm{B}$, and S are independent, the number represented by the output stochastic bit stream C is

$$
\begin{gather*}
c=P(C=1) \\
=P(S=1 \operatorname{and} A=1)+P(S=1 \operatorname{and} B=1)  \tag{2}\\
=P(S=1) P(A=1)+P(S=0) P(B=1) \\
=s \cdot a+(1-s) \cdot b
\end{gather*}
$$

Not only can SC perform arithmetic functions with a small number of gates, but it is also very noise resilient [17-19]. Some studies have shown that it can perform image filtering tasks even under conditions with $30 \%$ bit flips [17]. One of the key disadvantages of SC using CMOS based random number generators is the large number of transistors required to generate high quality random numbers. In some cases, random number generation can account for $80 \%$ of the total circuit area and total energy consumption [17]. One possible solution is to


Fig. 1. Diagram for (a) STT CRAM cell, (b) STT CRAM array, (c) SOT CRAM cell, and (d) SOT CRAM array.
TABLE I
CRAM LOGIC CRITERIA

| Logic operation | Preset output state | STT voltage criteria ( $\mathrm{V}_{\mathrm{B}}$ ) | SOT voltage criteria ( $\mathrm{V}_{\mathrm{B}}$ ) |
| :---: | :---: | :---: | :---: |
| BUFFER <br> NOT | $\begin{gathered} \text { AP } \\ \mathrm{P} \end{gathered}$ | $V_{C}^{S T}\left[\frac{R_{O}+R_{P}}{R_{O}}\right]<V_{B}<V_{C}^{S T}\left[\frac{R_{O}+R_{A P}}{R_{O}}\right]$ | $V_{C}^{S H}\left[\frac{R_{S H E}+R_{P}}{R_{S H E}}\right]<V_{B}<V_{C}^{S T}\left[\frac{R_{S H E}+R_{A P}}{R_{S H E}}\right]$ |
| $\begin{gathered} \text { AND } \\ \text { NAND } \end{gathered}$ | $\begin{gathered} \mathrm{AP} \\ \mathrm{P} \end{gathered}$ | $V_{C}^{S T}\left[\frac{R_{O}\left(R_{A P}+R_{P}\right)}{R_{A P} R_{P}+R_{O}\left(R_{A P}+R_{P}\right)}\right]<V_{B}<V_{C}^{S T}\left[\frac{2 R_{O} R_{A P}}{R_{A P}^{2}+2 R_{O} R_{A P}}\right]$ | $V_{C}^{S H}\left[\frac{R_{S H E}\left(R_{A P}+R_{P}\right)}{R_{A P} R_{P}+R_{S H E}\left(R_{A P}+R_{P}\right)}\right]<V_{B}<V_{C}^{S H}\left[\frac{2 R_{S H E} R_{A P}}{R_{A P}^{2}+2 R_{S H E} R_{A P}}\right]$ |
| $\begin{gathered} \text { OR } \\ \text { NOR } \end{gathered}$ | $\begin{gathered} \text { AP } \\ \mathrm{P} \end{gathered}$ | $V_{C}^{S T}\left[\frac{2 R_{O} R_{P}}{R_{P}^{2}+2 R_{O} R_{P}}\right]<V_{B}<V_{C}^{S T}\left[\frac{R_{O}\left(R_{A P}+R_{P}\right)}{R_{A P} R_{P}+R_{O}\left(R_{A P}+R_{P}\right)}\right]$ | $V_{C}^{S H}\left[\frac{2 R_{S H E} R_{P}}{R_{P}^{2}+2 R_{S H E} R_{P}}\right]<V_{B}<V_{C}^{S H}\left[\frac{R_{S H E}\left(R_{A P}+R_{P}\right)}{R_{A P} R_{P}+R_{S H E}\left(R_{A P}+R_{P}\right)}\right]$ |

$\mathrm{R}_{\mathrm{AP}(\mathrm{P})}=\mathrm{MTJ}$ resistance in the $\mathrm{AP}(\mathrm{P})$ state. $\mathrm{R}_{\mathrm{O}}=\mathrm{MTJ}$ resistance of preset state of output MTJ. $\mathrm{R}_{\mathrm{SHE}}=$ resistance of the spin Hall channel. $\mathrm{V}_{\mathrm{C}}{ }^{\mathrm{ST}}=\mathrm{STT}$ critical switching voltage. $\mathrm{V}_{\mathrm{C}}{ }^{\mathrm{SH}}=$ SOT critical switching voltage.
replace CMOS based random number generators with spintronic based ones. Previous studies have exploited the intrinsic stochasticity of magnetic tunnel junctions (MTJs) in order to generate stochastic bit-streams using a single nanodevice [20-21] rather than using a large number of transistors required for CMOS based random number generators. While these solutions significantly reduce the area cost of generating stochastic bits, external circuitry and additional computation steps are still required.

To avoid this short-coming, SC was implemented using the computational random access memory (CRAM) architecture, which is a spintronics based memory platform capable of performing logic operation directly within the memory cell [2223]. CRAM was first proposed in refs. [22-23] and demonstrated based on the STT-RAM array recently [24]. Performing SC operations via CRAM using a process called SC-CRAM has two key benefits over SC using CMOS based technologies. One is that the MTJ, which is a key component of the CRAM cell, has intrinsic stochasticity, which is an advantage for any MTJ based stochastic bit-generator. The second is that stochastic computation and random bit generation
can be done within the same memory cell in CRAM, thus eliminating the additional area cost and computation delay needed for CMOS based SC. In our previous work, we demonstrated that SC-CRAM outperformed conventional computing in CRAM in Local image thresholding, Bayesian inference, Bayesian belief network, and kernel density estimation in terms of computation delay, total circuit area, and noise margin [25].

While our previous results in SC-CRAM were promising, the overall performance could be improved. In particular, the energy consumption of SC-CRAM was higher than in conventional CRAM for all four neuromorphic applications examined. In this study, we investigate how the intrinsic properties of the MTJs affect the total energy consumption in SC-CRAM for various arithmetic functions and neuromorphic applications. Additionally, we compare the performance of SCCRAM between the spin transfer torque (STT) and spin orbit torque (SOT) switching mechanisms. STT-RAM [26] has been developed for decades and now commercially available from industry [27-30]. SOT-RAM has been proposed and developed in past ten years [31]. Promising aspects of SOT-RAM over


Fig. 2. (a) Illustration of task scheduling for the SC-CRAM process (read step not shown). (b) Example of switching probability distribution curves for projected STT MTJs at various pulse widths. Note that the switching probability distribution data is used to determine the proper pulse amplitude for the perturb step.

STT-RAM have been presented by many research groups, e.g. speed, endurance, reliability, and potential ultra-low writing energy [32-35]. Our calculations use key performance metrics from the experimental results from research labs, the current industry standards, and metrics for projected future MTJ devices for both STT and SOT switching. While this work focuses on the usage of MTJs for SC-CRAM with two different switching mechanisms, it should be noted that MTJs can be also tuned into the building blocks for probabilistic computing to address optimization problems [36-39]. This article is organized as follows. The background information on the CRAM architecture, the working principles of SC-CRAM, and the four neuromorphic computing applications tested are explained in section II. The calculations and benchmarking methods are explained in section III. The results of our calculations are presented in section IV and an analysis of the results obtained is discussed in section V. The article is then summarized in section VI.

## II.BACKGROUND

## A. SOT vs STT CRAM

Diagram for an STT- and SOT- CRAM cell is shown in Fig. 1(a) and 1(c), respectively. The configuration of these two cells have several similarities. Both use a 2-transistor/1 MTJ structure, which allows for separate logic and memory paths. This enables true in-memory computing capability in CRAM, where memory read and write operations are performed using
the memory word line (WL) and logic operations are performed within the CRAM cell by enabling the logic bit line (LBL). The difference between these two cells is that the write operations are done via the STT mechanism in Fig. 1(a) whereas the write operations are done via the SOT mechanism in Fig. 1(c).

During the logic operation, LBL is high, which allows for CRAM cells in the same row to be connected electrically through the logic line (LL). The examples illustrated in Fig. 1(b) and 1 (d) show an array of 3 cells in STT-CRAM and SOTCRAM, respectively. In both cases, two cells operate as input cells and one operates as the output cell. Voltage pulses ( $\mathrm{V}_{\mathrm{BSL}}$ ) are applied to the bit select lines (BSL) of the input cells, which generates a current through the output MTJ (IOUT), which is dependent on the resistance states of the input MTJs. The output MTJ will switch states if the voltage at the output MTJ exceeds the critical switching voltage $\left(\mathrm{V}_{\mathrm{C}}\right)$ of the output MTJ. For STTCRAM, the voltage of the output MTJ is $\mathrm{I}_{\text {OUT }} * \mathrm{R}_{\text {MTJ }}$, where $\mathrm{R}_{\text {MTJ }}$ is the resistance of the output MTJ and switching is done via STT. For SOT-CRAM, the Iout is applied to the spin Hall channel of the output cell, and the output voltage is Iout $* \mathrm{R}_{\text {She }}$, where $\mathrm{R}_{\text {SHE }}$ is the resistance of the spin Hall channel. The criteria for $\mathrm{V}_{\mathrm{BSL}}$ and the initial state of the output MTJ (output preset state) for various logic operations are shown in Table I.

## B. Working principles of SC-CRAM

The process for implementing stochastic computing in CRAM is called SC-CRAM. This process is unique to other stochastic computing schemes as it allows for stochastic bit generation and computation to be performed directly within the

TABLE II
CRITERIA FOR KEY ARITHMETIC FUNCTIONS IN SC-CRAM

| Function | Number of CRAM cells ${ }^{\text {(a) }}$ | Number of Computation steps ${ }^{\text {(b) }}$ | Equation | Logic gates used |
| :---: | :---: | :---: | :---: | :---: |
| Multiplication | 3 | 769 | $Y \approx A * B$ | AND |
| Scaled Addition | 9 | 1027 | $Y \approx S * A+(1-S) * B$ | MUX (1 NOT, 2 AND, 1 OR) |
| Scaled division | 8 | 1027 | $Y \approx \frac{A}{A+B}$ | JK Flip Flop (1 NOT, 1 BUFFER, 4 NAND) |
| Absolute valued subtraction ${ }^{\text {(c) }}$ | 8 | 1282 | $Y \approx\|A-B\|$ | XOR (1 NAND, 1 OR, 1 AND) |
| Square root | 11 | 1538 | $Y \approx \sqrt{X}$ | $1 \mathrm{AND}+2 \mathrm{OR}$ |
| Exponential | 19 | 3331 | $Y \approx \exp (-4 X)$ | 3 NAND, 2 AND + 4 cascading AND |

${ }^{(a)}$ Number of cells indicate the number of CRAM columns used in the sub-array. All functions only require a single row of CRAM cells.
${ }^{\text {(b) }}$ Assumes 8 bit resolution ( 256 length bit streams).
${ }^{(c)}$ Assumes maximum correlation between input bit-streams
same CRAM cells, therefore, external circuitry is not required for random number generation. The full process of SC-CRAM consist of synchronized reset, perturb, logic, and read cycles.

To illustrate how multiplication of two bit-streams, A and B, is done using AND logic in SC-CRAM, consider the example shown in Fig. 2(a). This figure outlines all the four cycles of SC-CRAM. During the reset step, voltage pulses, $\mathrm{V}_{\text {RES }}$, are applied along the memory bit-lines of each cell, which initializes the input MTJs to the P-state and the output MTJ, Y, to the AP-state. Note that the initial state of the MTJ depends on the function of the cell. Any cells that undergo the perturb step will be initialized to the $P$-state whereas any cells that serve as output cells during logic operations will be set to the proper preset state for the desired logic function (recall table I). Since the example in Fig. 2(a) is performing AND logic, the output MTJ is set to the AP-state.

During the perturb step, voltage pulses $\mathrm{V}_{P}{ }^{(\mathrm{A})}$ and $\mathrm{V}_{\mathrm{P}}{ }^{(\mathrm{B})}$ are applied along the memory bit lines of input cells A and B. Unlike the reset and logic steps, the voltage pulses during the perturb step cause the MTJs to switch probabilistically rather than deterministically. The amplitude and duration of $\mathrm{V}_{\mathrm{P}}{ }^{(\mathrm{A})}$ and $\mathrm{V}_{\mathrm{P}}{ }^{(\mathrm{B})}$ determine the switching probability of A and B , which can be obtained through switching probability distribution data from the MTJs, an example of which is shown in Fig. 2(b). There are three ways of determining the desired switching probability of A and B . One is through machine learning algorithms, however, this is only applicable in neural networks where the input cell being used sets a synaptic weight. The second is when performing functions involving a fixed constant, in which case, the switching probability needs to be equal to the desired value of the constant. The third method of determining the desired switching probabilities is when these values are dependent on the input data. For example, in image processing applications, the switching probability of A would be dependent on the intensity of the corresponding pixel.

The logic step in SC-CRAM follows the same criteria that is listed in table I. Since the example shown in Fig. 2(a) is illustrating AND logic, $\mathrm{V}_{\mathrm{B}}$ should be set so that Y switches to the P -state when $\mathrm{A}, \mathrm{B}$, or both are in the P -state, but Y should remain in the AP-state if both A and B are in the AP-state. Note that the resistance state of $A$ and $B$ are probabistic, however, switching during the logic step is still deterministic.

During the read step, the final state of Y is measured using a
voltage pulse $\mathrm{V}_{\mathrm{R}}$, which should be small enough so that it does not affect the resistance state of Y. It should be noted that some functions require multiple logic steps. For these functions, the read step is replaced by the proceeding logic operation. In this case, the output cell for the first logic function serves as the input cell for the second logic function. The read step for each function only needs to be performed at the output of the final logic operation.

## C. Arithmetic Functions in SC-CRAM

Figure 2(a) outlines the process for computing the multiplication of bit-streams A and B using AND logic. However, SC-CRAM is capable of performing a wide variety of arithmetic functions. In this study, we will investigate the performance of five additional functions, which are outlined in table II. These functions are scaled addition, scaled division, absolute valued subtraction, square root, and exponential. Note that these functions were chosen because they are important functions in the four applications being analyzed, which are described in further detail in section II. C. Also note that the SCCRAM process for these functions are also described in our previous work [25].

Scaled addition can be accomplished using MUX, which can be implemented using 2 AND gates, 1 NOT gate, and 1 OR gate. Note that in SC-CRAM, it is more feasible to perform NOT logic using a NAND gate, where 1 of the inputs is set to a constant value of ' 1 '. Furthermore, OR logic should be performed using 2 NOT gates and a NAND gate (see ref. [25]). The total circuit in SC-CRAM consists of 3 input cells (A, B, and S), 5 intermediate cells ( $\bar{S}, \mathrm{M}_{1}, \mathrm{M}_{2}, \overline{M_{1}}$, and $\overline{M_{2}}$ ), and 1 output cell (Y). First, perturb pulses are applied to A, B, and S, where the switching probabilities of $A$ and $B$ are input dependent and the switching probability of S is 0.5 . This is followed by one NOT logic step on S to obtain $\bar{S}$ and two AND logic steps on $\mathrm{A}, \mathrm{S}$ and $\mathrm{B}, \bar{S}$ to obtain $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$, respectively. Two additional NOT logic steps are applied to $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ to obtain $\overline{M_{1}}$, and $\overline{M_{2}}$ and finally, NAND logic is applied to obtain Y. The final output bit-stream should produce the function $\mathrm{Y} \approx$ $S^{*} \mathrm{~A}+(1-\mathrm{S}) * \mathrm{~B}$ or $\mathrm{Y} \approx 0.5 *(\mathrm{~A}+\mathrm{B})$.

Scaled division of stochastic bit-streams A and B can be calculated using the logic for a JK flip-flop. Note that inputs A and B correspond to the J and K terminals, respectively. The JK flip-flop is implemented using 1 NOT gate, 4 NAND gates, and

1 BUFFER. It should be noted that BUFFER logic in SCCRAM is the more feasible using an AND gate, where one of the inputs is set to a constant value of ' 1 '. The total circuit in SC-CRAM consists of 2 input cells (A and B), 5 intermediate cells $\left(\mathrm{Q}, \overline{\mathrm{Q}}, \mathrm{J}, \mathrm{K}_{1}, \mathrm{~K}_{2}\right)$, and 1 output MTJ (Y). Perturb pulses are applied to $A$ and $B$, which are input dependent. This is followed by one NOT step on Q to obtain $\overline{\mathrm{Q}}$, one NAND step on $\overline{\mathrm{Q}}$ and A to determine $J$, one NAND step on $Q$ and $B$ to determine $K_{1}$, and one NAND step on $Q$ and $K_{1}$ to obtain $K_{2}$. The final steps are one NAND step on $K_{2}$ and $J$ to determine $Y$ followed by one BUFFER operation on Y to obtain Q for the next cycle. Note Q is set to ' 0 ', or the P-state, for the first cycle. The bit-stream for $Y$ should produce the function $Y \approx A /(A+B)$.

Absolute valued subtraction of stochastic bit-streams A and $B$ can be done using XOR logic. In CRAM, an XOR gate can be implemented using 1 NAND gate, 1 AND gate, and one OR gate, which consists of 2 NOT gates and 1 NAND gate. The total circuit in SC-CRAM consists of 2 input cells (A and B), 4 intermediate cells $\left(\mathrm{M}_{1}, \mathrm{M}_{2}, \bar{A}\right.$, and $\left.\bar{B}\right)$, and 1 output cell $(\mathrm{Y})$. Perturb pulses are applied to A and B , which are input dependent, however, unlike the other functions, the bit-streams for A and B need maximum correlation. The process for achieving maximum correlation will be explained further in section II. C. The perturb step is followed by two NOT steps on A and B to obtain $\bar{A}$ and $\bar{B}$, one NAND step on A and B to obtain $\mathrm{M}_{1}$, one NAND step on $\bar{A}$ and $\bar{B}$ to obtain $\mathrm{M}_{2}$, and one AND logic step on $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ to obtain Y . The final output bitstream should produce the function $Y \approx|A-B|$.

The square root of stochastic bit-stream $X$ can be calculated using AND logic followed by two layers of OR logic. Note that the two OR gates both consist of two NOT gates and one NAND gate. The total SC-CRAM circuit consists of 4 input cells ( $\mathrm{X}_{1}$, $\underline{\mathrm{X}_{2}}, \mathrm{C}_{1}$, and $\mathrm{C}_{2}$ ), 6 intermediate cells ( $\overline{X_{2}}, \mathrm{M}_{1}, \mathrm{M}_{2}, \overline{M_{1}}, \overline{M_{2}}$, and $\overline{C_{2}}$ ) and 1 output MTJ (Y). Perturb pulses are applied to $\mathrm{X}_{1}, \mathrm{X}_{2}$, $\mathrm{C}_{1}$, and $\mathrm{C}_{2}$, where the switching probabilities of $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ are equal and input dependent and the switching probabilities of $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ are fixed at $67 \%$ and $18 \%$, respectively. Note that even though the switching probabilities of $X_{1}$ and $X_{2}$ are equal, the bit-streams should be generated independently, meaning they are decorrelated. This is followed by one AND step on $\mathrm{X}_{1}$ and $\mathrm{C}_{1}$ to obtain $\mathrm{M}_{1}$, two NOT logic steps on $\mathrm{M}_{1}$ and $\mathrm{X}_{2}$ to obtain $\overline{M_{1}}$ and $\overline{X_{2}}$, one NAND step on $\overline{M_{1}}$ and $\overline{X_{2}}$ to obtain $\mathrm{M}_{2}$, two NOT steps on $\mathrm{M}_{2}$ and $\mathrm{C}_{2}$ to obtain $\overline{M_{2}}$ and $\overline{C_{2}}$, and one NAND step on $\overline{M_{2}}$ and $\overline{C_{2}}$ to obtain Y. The final output bit-stream should produce the function $Y \approx \sqrt{ }\left(\mathrm{X}_{1}\right)$.

The method of solving an exponential function for our analysis was the same as the one described in ref. [15]. In our case, our desired function was $Y \approx \exp (-4 x)$. Note that this function can be re-written as $Y \approx \exp [(-4 / 5 x) 5]$. The first step is to solve $\exp (-4 / 5 x)$ using the $3^{\text {rd }}$ order Maclaurin expansion. This can be implemented in SC-CRAM using 3 NAND gates and 2 AND gates. The total SC-CRAM circuit for this first step consists of 6 input cells $\left(\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}, \mathrm{~A}_{1}, \mathrm{~A}_{2}\right.$, and $\left.\mathrm{A}_{3}\right), 4$ intermediate cells $\left(M_{1}, M_{2}, M_{3}\right.$, and $\left.M_{4}\right)$, and 1 output cell $\left(B_{0}\right)$. Perturb pulses are applied to all the input cells, where the switching probabilities of $X_{1}, X_{2}$, and $X_{3}$ are all equal, decorrelated, and input dependent and the switching probabilities of $A_{1}, A_{2}$, and $A_{3}$ are fixed at $80 \%, 40 \%$, and
$26.7 \%$, respectively. NAND logic is used on $X_{1}$ and $A_{3}$ to obtain $M_{1}$, AND logic is used on $M_{1}$ and $A_{2}$ to obtain $M_{2}$, NAND logic is used on $\mathrm{M}_{2}$ and $\mathrm{X}_{2}$ to obtain $\mathrm{M}_{3}$, AND logic is used on $\mathrm{M}_{3}$ and $A_{1}$ to obtain $M_{4}$, and NAND logic is used on $M_{4}$ and $X_{3}$ to obtain $B_{0}$. Note that the bit-stream for $B_{0}$ is approximately $\exp (-$ $4 / 5 x)$. In order to calculate $\exp (-4 x)$ from $B_{0}, 4$ additional cascading AND gates are used on the bit-stream for $\mathrm{B}_{0}$, where $\mathrm{B}_{0}$ is buffered at each gate.

## D. Neuromorphic computing applications

In this section, we describe the four applications that were analyzed in this study. These applications are Local image thresholding for character recognition, Bayesian inference for object location, Bayesian belief network for heart disaster prediction, and Kernel density estimation for object recognition. For each application, we compare the energy consumption and noise margin between each category of MTJs. It should be noted that the applications were also analyzed in our previous work [25], where we compared the performance metrics of SC-CRAM to conventional CRAM. Our results showed that SC-CRAM required significantly less CRAM cells than conventional CRAM, which leads to improved noise margin. Additionally, SC-CRAM requires less computation steps to perform these tasks, which can be attributed to the large number of logic gates required for conventional CRAM. Furthermore, the perturb and reset steps can be performed in parallel with the logic steps in separate cells, meaning that the computation speed of SC-CRAM does not increase as drastically with network size as it does with conventional CRAM. In this study, we are comparing the performance metrics of different MTJ technologies.

Image thresholding is a very important step for optical character recognition. M. Najafi and M. Salehi applied SC to a local image threshold technique called the Sauvola method [19]. In this method, a window of $9 \times 9$ pixels is defined within a sub-section of a degraded input image. At each sub-section, a threshold (T) is calculated using (1a), where $A(x, y)$ is the pixel intensity at point (x,y) and $\bar{A}$ and $\sigma \mathrm{A}$ are the mean and standard deviation of all the pixels within the window and $\sigma \mathrm{A}$ is calculated using (1b). The circuit for calculating (1a) is shown in Fig. 3(a). Note that XOR logic is used to find $\left|\overline{A^{2}}-(\bar{A})^{2}\right|$. In this example, maximum correlation between the bit-streams for $\overline{A^{2}}$ and $(\bar{A})^{2}$ can be ensured since the bit-streams for A are used to generate bitstreams for $\mathrm{A}^{2}$.

$$
\begin{gather*}
T(x, y)=\bar{A}(x, y) \cdot\left(\frac{\sigma A(x, y)+1}{2}\right)  \tag{1a}\\
\sigma A(x, y)=\sqrt{\left|\overline{A^{2}}-(\bar{A})^{2}\right|} \tag{1b}
\end{gather*}
$$

In the second application, a Bayesian inference system is used to determine the location of an object using distance (D) and bearing (B) data from three noisy sensors. The Bayesian inference mechanism, which is described in further detail in ref [36], produces a distribution of object locations by calculating the product series of conditional probabilities. The models for the probabilities for D and B for sensor j at position ( $\mathrm{x}, \mathrm{y}$ ) can be described the Gaussian distributions in (2a) and (2b), where


Fig. 3. Circuit diagrams for (a) local image thresholding (LIT), (b) Bayesian inference system (BIS), (c) Bayesian belief network (BBN), and (d) Kernel density estimation (KDE). Images extracted from ref. [25].
$\mu_{\mathrm{dj}}$ is the Euclidean distance between sensor j and position ( $\mathrm{x}, \mathrm{y}$ ), $\theta_{\mathrm{dj}}$ is equal to $5+\mu_{\mathrm{dj}} / 10, \mu_{\mathrm{bj}}$ is the viewing angle of sensor j , and $\theta_{\mathrm{bj}}$ is set to $14.0626^{\circ}$. This data is used to calculate the object location probability at position ( $\mathrm{x}, \mathrm{y}$ ), $\mathrm{p}(\mathrm{x}, \mathrm{y})$, using (2c). The problem described in [40] and used for our analysis $p(x, y)$ data on a $64 \times 642 \mathrm{D}$ grid and the sensors are located at positions $(0,0),(0,32)$, and $(32,0)$. The circuit to solve (2c) is shown in Fig. 3(b). Each grid location uses 5 AND gates to calculate $\mathrm{p}(\mathrm{x}, \mathrm{y})$. When implemented in SC-CRAM 11 cells are needed at each location ( 3 at the input AND gate and 2 for the remaining 4 layers).

$$
\begin{gather*}
p\left(D_{j} \mid x, y\right)=\mathrm{N}\left(\mu_{d j}, \theta_{d j}\right)  \tag{2a}\\
p\left(B_{j} \mid x, y\right)=\mathrm{N}\left(\mu_{b j}, \theta_{b j}\right)  \tag{2b}\\
p(x, y)=\prod_{j} p\left(B_{j} \mid x, y\right) * p\left(D_{j} \mid x, y\right) \tag{2c}
\end{gather*}
$$

The third application is a Bayesian belief network (BBN) for heart disaster (HD) prediction, which is a probabilistic graphical model that represents a set of random variables and their conditional dependencies via a directed acyclic graph. The parent nodes in this network are exercise (E) and diet (D) and the child nodes are high blood pressure (BP) and chest pain $(\mathrm{CP})$. The conditional probability tables for each node are shown in refs [40]. The probability of a heart disaster, $\mathrm{P}(\mathrm{HD})$, is shown in (3a), where $\mathrm{P}_{\mathrm{HD}}{ }^{\mathrm{E}, \mathrm{D}}$ is the heart disaster probability when only considering E and $\mathrm{D}, \mathrm{P}_{\mathrm{BP}}$ and $\mathrm{P}_{\mathrm{CP}}$ are the heart disaster probabilities for cases of high blood pressure and chest pain, respectively. The expression for $\mathrm{P}_{\mathrm{HD}}{ }^{\mathrm{E}, \mathrm{D}}$ is shown in (3b),
where $P_{D}, P_{E}$, and $P_{E, D}$ represent the heart disaster probabilities for cases of regular exercise, a good diet, and both, respectively. The circuit for calculating (3a) is shown in Fig. 3(c). Note that the final value for $\mathrm{P}(\mathrm{HD})$ is calculated using a JK flip-flop, which was implemented in SC-CRAM using the process described in the previous section.

$$
\begin{gather*}
p(H D)=\frac{P_{B P} * P_{C P} * P_{H D}^{E, D}}{P_{B P} * P_{C P} * P_{H D}^{E, D}+\overline{P_{B P}} * \overline{P_{C P}} * \overline{P_{H D}^{E, D}}}  \tag{3a}\\
p_{H D}^{E, D}=\left[P_{E, D} P_{D}+P_{E, \bar{D}} P_{\bar{D}}\right] P_{E}+\left[P_{\bar{E}, D} P_{D}+P_{\bar{E}, \bar{D}} P_{\bar{D}}\right] P_{\bar{E}} \tag{3b}
\end{gather*}
$$

The fourth application examined is Kernel Density Estimation (KDE) which is an image segmentation algorithm for object recognition, surveillance, and tracking [18]. The KDE algorithm is based on recent information that is continuously updating. Sample values of pixel intensity, X, are captured from recent iterations $\left(\mathrm{X}_{\mathrm{t}}, \mathrm{X}_{\mathrm{t}-1}, \ldots, \mathrm{X}_{\mathrm{t}-\mathrm{N}}\right)$, which can be used to determine the probability density function (PDF), as described in (4). The circuit for calculating (4) is shown in Fig. 3(d), where $X_{t}$ represents the pixel intensity at time $t$ and $X_{t-i}$ represents the pixel intensity at the $i^{\text {th }}$ previous cycle.

$$
\begin{equation*}
\operatorname{PDF}\left(X_{t}\right)=\frac{1}{N} \sum_{i=1}^{N} e^{-4\left|X_{t}-X_{t-i}\right|} \tag{4}
\end{equation*}
$$

## III. Calculations and Benchmarking

For our analysis, we considered the performance of SCCRAM for six different categories of MTJs. These are the top

STT and SOT performing MTJs from research groups, STT and SOT from industry, and projected performance of STT and SOT MTJs. For research MTJs, the parameters for STTCRAM, were obtained from refs. [41-44] and the parameters for SOT-CRAM were obtained from [34, 48-51]. For industry MTJs, the parameters for STT-CRAM and SOT-CRAM were obtained from those recently reported by IBM [45] and IMEC [52-53], respectively. For projected MTJs in STT-CRAM, we used the parameters that were predicted in refs [46-47]. However, since SOT-MRAM is not as well established for industrial purposes as STT-MRAM, accurate projected parameters for SOT MTJs are not yet reported. Therefore, for projected MTJs in SOT-CRAM, we used the best reported parameters from various different studies [35, 54].

For each category, we calculated the total energy consumption for six different arithmetic functions. These are multiplication, scaled addition, scaled division, absolute valued subtraction, square root, and exponential function. Recall that the logic gates used for each of these calculations were described in section II. B and are shown in table II. Total energy consumption was also calculated for each of the four neuromorphic computing applications described in section II. C. Lastly, our analysis considered the influence of variations in device dimensions and determined which category of MTJs was the most resilient to these random variations in SCCRAM. Details of which parameters we considered, how energy consumption was determined, and how the effect of device variations were implemented in our calculations are described in more detail in the following sub-sections.

## A. Parameters for analysis

There are six key MTJ properties that were considered for our analysis. These properties are the resistance-area (RA) product, the tunneling magnetoresistance (TMR) ratio, the thermal stability factor $(\Delta)$, the intrinsic critical switching current density ( $\mathrm{J}_{\mathrm{C} 0}$ ), the switching time $\left(\tau_{\mathrm{Sw}}\right)$, and the precessional switching coefficient $\left(\mathrm{A}_{\mathrm{v}}\right)$. For all of our calculations, we assumed that the MTJ pillars were patterned into circular nanopillars with a diameter of 20 nm , meaning that the area is approximately $314 \mathrm{~nm}^{2}$. Therefore, the resistances in the AP and P-state ( $\mathrm{R}_{\mathrm{AP}}$ and $\mathrm{R}_{\mathrm{P}}$, respectively) can be calculated from the RA product and the TMR ratio, where $R_{P}=R A /$ area and TMR $=100 *\left[\left(\mathrm{R}_{\mathrm{AP}}-\mathrm{R}_{\mathrm{P}}\right) / \mathrm{R}_{\mathrm{P}}\right]$. It should be noted that the lowest value recorded of JC0 for SOT switching is

In our calculations, $\tau_{\text {SW }}$ is determined to be the perturb pulse width where the switching energy is minimized, which will be explained in further detail in the next section. When operating in the precessional switching regime $\left(\tau_{\mathrm{Sw}}<5 \mathrm{~ns}\right), \mathrm{A}_{\mathrm{V}}$ determines the relation between pulse width and pulse amplitude, where $\tau_{\mathrm{Sw}}{ }^{-1}=\mathrm{A}_{\mathrm{V}}{ }^{*}\left(\mathrm{~V}-\mathrm{V}_{\mathrm{C} 0}\right)$. In some studies, the values for $A_{V}$ were explicitly reported and for other studies, we had to determine $A_{v}$ based on the data provided. In either case, values provided for $\mathrm{A}_{\mathrm{V}}$ assume a switching probability of $50 \%$. This is an accurate assumption when determining $\tau_{\text {SW }}$ for the perturb step, however, the actual value for $A_{v}$ may be different for the reset and logic steps since the desired switching

TABLE III
PARAMETERS USED FOR ANALYSIS

| STT-CRAM |  |  |  |
| :---: | :---: | :---: | :---: |
| Parameter | Research | Industry | Projected |
| RA product $\left(\Omega \cdot \mu \mathrm{m}^{2}\right)$ | 5 [37] | 3.68 [41] | 1 [42] |
| TMR ratio | 133 [38] | 82 [41] | 200 |
| $\Delta$ | 60 [39] | 45 [41] | 75 [43] |
| $\mathrm{J}_{\mathrm{C} 0}\left(\mathrm{MA} / \mathrm{cm}^{2}\right)$ | 3.1 [37] | 1.25 [41] | 1 [37] |
| $\tau_{\text {sw }}(\mathrm{ns})$ | 1.25 | 0.75 | 0.75 |
| $A_{V}\left(s^{-1} V^{-1}\right)$ | $2.1 \times 10^{9}$ [40] | $1.5 \times 10^{10}[41]$ | $1.5 \times 10^{10}[41]$ |
| SOT-CRAM |  |  |  |
| RA product $\left(\Omega \cdot \mu \mathrm{m}^{2}\right)$ | 12.3 [44] | 17.5 [48] | 1 [42] |
| TMR ratio | 94 [45] | 110 [49] | 200 |
| $\Delta$ | 45 [45] | 48 [49] | 60 [50] |
| $\mathrm{J}_{\mathrm{C} 0}\left(\mathrm{MA} / \mathrm{cm}^{2}\right)$ | 75 [46] | 100 [48] | 1 [35] |
| $\tau_{\text {SW }}(\mathrm{ns})$ | 2 | 0.75 | 0.25 |
| $A_{V}\left(s^{-1} V^{-1}\right)$ | $4.76 \times 10^{8}$ [48] | $1.46 \times 10^{10}$ [48] | $1.46 \times 10^{10}$ [48] |
| $\begin{aligned} & \text { SOT material/ } \\ & \rho(\mu \Omega \cdot \mathrm{cm}) \end{aligned}$ | Ta / 190 [34] | W / 160 [49] | BiSe / 2150 [50] |
| $\theta_{\text {SH }}$ | -0.25 [34] | -0.32 [48] | 2.88 [50] |
| $\mathrm{t}_{\text {SOT }}(\mathrm{nm})$ | 5 [47] | 3.5 [49] | 8 [50] |

probability is close to $100 \%$. Therefore, for the STT MTJs and research SOT MTJs, pulse widths of 5 ns were used for the reset and logic steps since the calculated energy consumption was minimized at this pulse width.

There are three additional parameters that we considered for our SOT-CRAM calculations. These are the resistivity of the spin Hall channel $(\rho)$, the spin Hall angle $\left(\theta_{\mathrm{SH}}\right)$, and the thickness of the spin Hall channel ( $\mathrm{t}_{\text {sот }}$ ). The values for $\rho$ were determined based on the material used for the spin Hall channel for each category. These materials were Ta, W, and BiSe for research MTJs, industrial MTJs, and projected MTJs, respectively. BiSe was chosen for projected MTJs since it has the largest $\theta_{\text {SH }}$ reported [54]. The values for $t_{\text {sot }}$ were chosen based on the spin Hall channel thickness that provided the maximum $\theta_{\text {SH }}$ for each material. For our calculations in SOTCRAM, we assumed a spin Hall channel width and length of 40 nm and 120 nm , respectively. From these values, we determined the resistance of the spin Hall channel ( $\mathrm{R}_{\text {SHE }}$ ) using the calculation $\mathrm{R}_{\text {SHE }}=\left(\rho^{*}\right.$ length $) /($ tsor $*$ width $)=3 \rho /$ tsot .

## B. Voltage and energy consumption calculations

For each category of MTJs, the total energy consumption was calculated for each arithmetic function described in table II. The energy consumed from a single voltage pulse is calculated from (5), where $\mathrm{R}_{\mathrm{MTJ}}$ is the MTJ resistance, V is voltage pulse amplitude and $t_{P}$ is the pulse width. Note that $R_{M T J}$ is either $R_{A P}$ or $R_{P}$, depending on the state of the initial state of the MTJ and for SOT switching, $\mathrm{R}_{\text {MTJ }}$ is replaced by $\mathrm{R}_{\text {SHE }}$. The total energy consumption ( $\mathrm{E}_{\mathrm{TOT}}$ ) is determined by (6), where $\mathrm{N}_{\mathrm{B}}$ is the total


Fig. 4. Plot for (a) switching energy vs pulse width and (b) pulse amplitude vs. pulse width for the perturb operation in research MTJs.
number of bits and $E_{\text {PERT }}, \mathrm{E}_{\text {LOGIC }}$, and $\mathrm{E}_{\text {RES }}$ are the energies during the perturb, logic, and reset step respectively. For our calculations, we assumed 8-bit resolution, therefore, $\mathrm{N}_{\mathrm{B}}=256$.

$$
\begin{gather*}
E=\frac{V^{2} t_{P}}{R_{M T J}}  \tag{5}\\
E_{T O T}=N_{B} *\left(E_{P E R T}+E_{L O G I C}+E_{R E S}\right) \tag{6}
\end{gather*}
$$

For all six categories of MTJs, three different combinations of V and $\mathrm{t}_{\mathrm{p}}$ were determined, one for perturb, another for logic, and a third for reset operations. To calculate these values, we first considered the desired switching probability $\left(\mathrm{P}_{\mathrm{SW}}\right)$, as defined in (7), where $\tau$ is the characteristic switching time. For thermal activation switching ( $\mathrm{t}_{\mathrm{P}} \geq 5 \mathrm{~ns}$ ), $\tau$ is defined by (8) where $\tau_{0}$ is the inverse attempt frequency, which we assumed to be 1 ns [55] and $\mathrm{V}_{\mathrm{C} 0}$ is the intrinsic switching voltage, which was directly calculated from $\mathrm{J}_{\mathrm{C} 0}$ using $\mathrm{V}_{\mathrm{C} 0}=\left(\mathrm{J}_{\mathrm{C} 0} /\right.$ area $) * \mathrm{R}_{\mathrm{MTJ}}$.

$$
\begin{align*}
& P_{S W}=1-\exp \left(-\frac{t_{P}}{\tau}\right)  \tag{7}\\
& \tau=\tau_{0} \exp \left[\Delta\left(1-\frac{V}{V_{C 0}}\right)\right] \tag{8}
\end{align*}
$$

It should be noted that (8) cannot be used for precessional switching ( $\mathrm{t}_{\mathrm{P}}<5 \mathrm{~ns}$ ). For the perturb step, we assumed $\mathrm{P}_{\mathrm{SW}}=$ 0.5 , which means that $t_{P} \approx \tau$. This means that V could be calculated directly from $t_{P}$ using $t_{P}{ }^{-1}=A_{V} *\left(V-V_{C 0}\right)$. However, for the reset and logic steps, we assumed $\mathrm{P}_{\mathrm{SW}}=0.99$, so this calculation needs to be modified. From (5), we can determine that $\mathrm{t}_{\mathrm{P}} / \tau \approx 4.6$ when $\mathrm{P}_{\mathrm{SW}}=0.99$, meaning that $\tau$ can be used to determine V . This means that AV should be modified


Fig. 5. Output probability vs input probability for $0 \%$ and $20 \%$ variations in MTJ diameter. This example shows multiplication of two input bit-streams via AND logic in SC-CRAM for research STT MTJs.
accordingly for cases when $\mathrm{P}_{\mathrm{SW}}=0.99$. Note that this modification of $A_{V}$ was only done for the logic and reset steps but was not necessary for perturb steps.

We calculated V from $\mathrm{t}_{\mathrm{p}}$ values ranging between 0.25 ns to 20 ns for perturb, reset, and logic operations and calculated the switching energy at each point using (5). The combination of V and $t_{p}$ that provided the lowest switching energy was the one that was chosen in our calculations. The example provided in Fig. 4(a) shows a plot of the switching energy vs. $t_{p}$ for the perturb operation in the research STT MTJs. Here we see that the switching energy is minimized when $t_{p}=1.25 \mathrm{~ns}$, which corresponds to $\mathrm{V}=544 \mathrm{mV}$, as shown in Fig. 4(b). This calculation was repeated for logic and reset operations as well as for all operations for the remaining 5 MTJ categories.

## C. Device variations

Our calculations also considered the effect of device variations to see which category of MTJs were the most resilient to noise and imperfections in SC-CRAM operations. In our analysis, device variations were measured in terms of a percent change in MTJ diameter from the nominal 20 nm . For SOT-CRAM, we also had to account for percent change in spin Hall channel width from the nominal 40 nm . Note that a percent change in MTJ diameter directly correlates to an equal percent change in $R_{P}$ and $R_{A P}\left(\sigma R_{P}\right.$ and $\sigma R_{A P}$, respectively). Furthermore, since $\Delta$ and $V_{\mathrm{C} 0}$ are proportional to the free layer volume, we assumed that $\sigma \Delta=-\sigma \mathrm{R}_{\mathrm{MTJ}}$ and $\sigma \mathrm{V}_{\mathrm{C} 0}=0.1 * \sigma \mathrm{R}_{\mathrm{MTJ}}$.

The SC-CRAM process for the six arithmetic functions listed in table II were simulated in MATLAB for bit-stream probabilities ranging from $10 \%$ to $90 \%$ for all six MTJ categories. Each simulation was repeated 100 times and the final calculation was averaged among all trials. To quantify the accuracy of the output, the mean square error (MSE) was calculated from these simulations for all six categories of MTJs considered. Expression for MSE is shown in (9), where NPTS is the total number of points plotted and $\mathrm{Y}_{\text {CALC }}$ and $\mathrm{Y}_{\text {EXP }}$ are the calculated and expected outputs, respectively. In our study we repeated each calculation for $\sigma \mathrm{R}_{\mathrm{MTJ}}=0-30 \%$, assuming a linear distribution. Note that $\mathrm{R}_{\text {MTJ }}$ was recalculated for each trial for each value of $\sigma \mathrm{R}_{\mathrm{MTJ}}$ tested. Figure 5 provides an example


Fig. 6. MSE vs MTJ dimension variation for (a) multiplication, (b) scaled addition, (c) scaled division, (d) absolute valued subtraction, (e) square root, and (f) exponential function in SC-CRAM. The results for the STT and SOT MTJs are shown as solid and dotted lines, respectively. The lines for the research, industrial, and projected MTJs are shown with blue circles, red triangles, and black squares, respectively.
for output probability vs input probability for multiplication via AND logic in SC-CRAM using research STT MTJs for 0\% noise and $20 \%$ noise, which illustrates how MSE was calculated.

$$
\begin{equation*}
M S E=\frac{1}{N_{P T S}} \sum_{i=1}^{N_{P T S}}\left(Y_{E X P, i}-Y_{C A L C, i}\right)^{2} \tag{9}
\end{equation*}
$$

## IV. Results

## A. Influence of device variations

Figures 6(a) - 6(f) show the results for MSE calculations for MTJ device variations ( $\sigma \mathrm{R}_{\text {MTJ }}$ ) between $0 \%$ to $30 \%$. MSE for multiplication via AND logic is shown in Fig. 6(a). These results show that the projected SOT and STT MTJs are the most resilient to MTJ variations, where $\mathrm{MSE}<10^{-4}$ even when $\sigma \mathrm{R}_{\text {MTJ }}$ $=30 \%$. Research SOT and STT MTJs will maintain an acceptable MSE $<10^{-3}$ when $\sigma \mathrm{R}_{\text {MTJ }}<20 \%$. However, the industry STT MTJs had the worst performance at both low and high $\sigma \mathrm{R}_{\mathrm{MTJ}}$. Recall that the projected SOT and STT MTJs both have a TMR ratio of $200 \%$ whereas the industry STT MTJs have a TMR ratio of $84 \%$. These results show that the accuracy and resiliency to device variations for multiplication using the AND logic are highly dependent on the TMR ratio.

The MSE calculations for scaled addition via MUX are shown in Fig. 6(b). These results show that the MSE is low when $\sigma \mathrm{R}_{\text {MTJ }}$ is low, but increases rapidly as $\sigma \mathrm{R}_{\text {MTJ }}$ increases.

This means that the results for scaled addition are more susceptible to variations than for multiplication. One possible explanation is that the output accuracy is dependent on the accuracy of the bit-stream generated from the selector (S) MTJ. The accuracy of the output will vary if output probability of S deviates from $50 \%$. However, the MSE is still lowest for projected SOT and STT MTJs, where MSE $<10^{-3}$ for $\sigma \mathrm{R}_{\text {MTJ }}$ up to $20 \%$.

Figure 6(c) shows that for scaled division via a JK flip flop, the MSE $<10^{-4}$ for $\sigma \mathrm{R}_{\text {MTJ }}$ up to $10 \%$ or $15 \%$ for all categories except for industry STT MTJs. Furthermore, the research and projected STT MTJs show that most resiliency for $\sigma \mathrm{R}_{\mathrm{MTJ}}<20 \%$. However, the MSE spikes above $10^{-3}$ for $\sigma R_{\text {MTJ }}=20 \%$. These results illustrate that device variations have a much stronger impact on accuracy as the number of consecutive NAND gates increases.

Figure 6(d) shows that the behavior of MSE with $\sigma \mathrm{R}_{\mathrm{MTJ}}$ for absolute valued subtraction via XOR logic is similar to that of scaled division. These results show that industry and research STT MTJs are the most susceptible to device variations. For the other 4 categories are less than $10^{-3}$ for $\sigma R_{\text {MTJ }}<25 \%$. Furthermore, the projected STT MTJs were the most resilient to device variations.

The MSE calculations for the square root function are shown in Fig. 6(e). These results show that the MSE values are higher for the square root function than any other functions. This can


Fig. 7. Energy consumption for the six arithmetic functions analyzed for all MTJ categories.
be attributed to the fact that the square root function relies on two MTJs that generate constant bit-streams, $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$, with probabilities of $18 \%$ and $67 \%$, respectively. As with the scaled addition function, the accuracy of the output will vary when the output probabilities of $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ deviate from their desired values. At low $\sigma \mathrm{R}_{\mathrm{MTJ}}$, the research STT MTJs have the lowest MSE, however, the projected STT and SOT MTJs are the most resilient to MTJ variations where $\mathrm{MSE}<10^{-3}$ for $\sigma \mathrm{R}_{\mathrm{MTJ}}=15 \%$.

Figure $6(\mathrm{f})$ shows the results of MSE with $\sigma \mathrm{R}_{\mathrm{MTJ}}$ for the exponential function. For most of the MTJ categories, MSE increased steadily with $\sigma \mathrm{R}_{\text {MTJ }}$ from $10^{-5}$ for $\sigma \mathrm{R}_{\text {MTJ }} \leq 10 \%$ to MSE $\approx 10^{-3}$ for $\sigma \mathrm{R}_{\mathrm{MTJ}}=30 \%$. The two exceptions are the STT research MTJs and the projected SOT MTJs. The research STT MTJs showed the highest MSE for all $\sigma \mathrm{R}_{\text {MTJ }}$ values. Alternatively, the projected SOT MTJs showed the best resiliency to device variations, where $\mathrm{MSE} \approx 10^{-4}$ at $\sigma \mathrm{R}_{\mathrm{MTJ}}=$ $30 \%$. It should be noted that the exponential function has more perturb steps than the other five functions analyzed. Therefore, these results indicate that calculations in SC-CRAM have the best resiliency to variations in switching probability when using the projected SOT MTJs and the worst resiliency when using the research STT MTJs

## B. Comparison of energy consumption

The bar plots in Fig. 7 show the energy consumption for each function in each MTJ category. These plots show that for all categories, multiplication consumes the least amount of energy and the exponential function consumes the most energy at around 10X more than multiplication. Scaled addition, scaled division and absolute valued subtraction functions all consume more energy than multiplication but less than the exponential function. This trend does not differ significantly between each MTJ category because it reflects the number of CRAM cells used and the number of logic steps required for each function.

The energy consumption for STT research MTJs is about one order of magnitude larger than for STT industrial and projected MTJs for all functions. This can be attributed to both larger $\mathrm{J}_{\mathrm{C} 0}$
and larger RA product for research STT MTJs when compared to industrial and projected STT MTJs. Both of these factors lead to larger voltages required for the perturb, logic, and reset steps. Additionally, the energy consumption for the industrial and projected STT MTJs is reduced further in comparison to the research STT MTJs by the fact that their switching energy is minimized at shorter pulses (recall table III).

The results in Fig. 7 also shows that the energy consumption for research SOT MTJs is approximately 3X larger than for industrial SOT MTJs. This may seem counterintuitive since $\mathrm{J}_{\mathrm{C} 0}$ is larger for the industrial SOT MTJs. However, there are two factors that cause the energy consumption in research SOT MTJs to increase when compared to industrial SOT MTJs. One is that the industrial SOT MTJs use shorter pulse widths for switching (recall table III) and the second is the smaller RA product in the MTJ pillars for the industrial MTJs. The projected SOT MTJs had the lowest energy consumption of the SOT categories, where the energy consumption was over one order of magnitude lower than the industrial SOT MTJs over two orders of magnitude lower than the research SOT MTJs.

One key observation is that the energy consumption in the STT MTJs is significantly lower than the research and industrial SOT MTJs. There are two factors that cause this increase in energy consumption for the research and industrial SOT MTJs. One is the larger $\mathrm{J}_{\mathrm{C} 0}$ values, which leads to larger voltages for the perturb and reset steps. The second, and much more significant factor is the large RA products of the MTJ pillars for the research and industrial SOT MTJs. This increases the voltage required for the logic step since the resistance of the pillars is much larger than the resistance of the spin Hall channel. However, the energy consumption in the projected SOT MTJs is lower than any of the STT MTJ categories. This is because the logic voltages are reduced significantly for projected SOT MTJs because the RA product is much lower and the resistance of the BiSe spin Hall channel is much larger when compared to the research and projected SOT MTJs. For optimal performance of SOT-based SC-CRAM, the resistance of the


Fig. 8. Energy consumption for the four neuromorphic computing applications analyzed for all MTJ categories.
spin Hall channel should be equivalent to the resistance of the MTJ pillar. Furthermore, low RA product of the MTJ pillars is desired.

## C. Performance metrics in neuromorphic applications

Figure 8 shows the total energy consumption for local image thresholding, object location, heart disaster prediction, and kernel density estimation for all MTJ categories. It should be noted that the analysis for number of CRAM cells and computation steps in SC-CRAM was done in our previous work [25] and will not change between the MTJ categories. The bar plots in Fig. 8 show that for each MTJ category, local image thresholding has the highest energy consumption followed by kernel density estimation, heart disaster prediction, and object location. This is simply a reflection on the number of CRAM cells required for each task. Furthermore, the trends observed in Fig. 8 follow similar trends to those observed in Fig. 7, where the research and industrial SOT MTJs have the largest energy consumption among the MTJ categories and the projected SOT MTJs have the lowest.

One of the key observations from the results in Fig. 8 is that relative energy consumption for local image thresholding and kernel density estimation between SOT and STT MTJs is lower than the results in Fig. 7. In Fig. 7, the energy consumption for the projected SOT MTJs is approximately $1.05-1.3 \mathrm{X}$ lower than for projected STT MTJs. However, in Fig. 8, the energy consumption for projected SOT MTJs is approximately $1.5-$ 1.6X lower than for projected STT MTJs for local image thresholding and kernel density estimation. This is because both of these tasks require a large number of NAND logic operations. For SOT MTJs, the energy consumption for NAND logic is the same for AND logic. However, for STT MTJs, the energy consumption for NAND logic is larger than for AND logic. The energy consumption is similar between STT and SOT MTJs for most arithmetic functions. However, these results show that the energy consumption in projected SOT MTJs decreases relative
to the projected STT MTJs for larger scale tasks, particularly those involving NAND logic.

## V.DISCUSSION

## A. Explanation of key results

The two sources of error that contribute to the increase in MSE with $\sigma \mathrm{R}_{\text {MTJ }}$ shown in Fig. 6(a-f) are errors in the logic operation and inaccurate switching probabilities during the perturb step. With no MTJ variations, the MSE should be minimal $\left(<10^{-5}\right)$ since the only source of error are small changes in switching probability. Note that this can be improved by increasing the bit-stream length. However, MTJ variations will change their properties, meaning that $\mathrm{V}_{\mathrm{B}}$ may fall outside the range of values outlined in table I. This will lead to logic errors since $V_{B}$ will either be too small and fail to switch the output MTJ for the proper input configuration or too large and will switch the MTJ for incorrect input configurations. The range of $\mathrm{V}_{\mathrm{B}}$ values for proper logic operations in CRAM increases as the TMR ratio of the MTJs increases. This explains why the projected SOT and STT MTJs were the most resilient to MTJ variations for most of the functions in Figs. 6(a) - 6(f) and the industry MTJs were the most sensitive to these variations.

Perturb errors are relatively insignificant compared to logic errors, however, there are a few functions where perturb errors have a larger impact. Namely, scaled addition and square root function are more influenced by perturb errors. This is because these two functions rely on bit-streams that are generated separately from the input bit-streams with a constant, predetermined probability. Not only does this add additional sources of perturb errors, but it also changes the function being processed. For example, the square root function uses constant probabilities of $18 \%$ and $67 \%$ to process the function $Y \sim X^{0.5}$. Changing the constant probabilities will change the function being processes.

Another key result shown in the data in Fig. 6(a-f) is that variations had a larger influence on SOT MTJs than STT MTJs.


Fig. 9. Breakdown of energy consumption between the reset, perturb, and logic steps for multiplication via AND logic for all MTJ categories.

Note that projected SOT MTJs have more resilience to variations than industry MTJs, they do not have more resilience to projected STT MTJs for most functions. In some cases, the research STT MTJs had better resilience to variations than the projected SOT MTJs. This is because, for SOT MTJs, device variations influence the diameter of the MTJ pillar as well as the width of the spin Hall channel whereas for STT MTJs, device variations only influence the diameter of the MTJ pillar. This additional source of variation for SOT MTJs means that $\mathrm{V}_{\mathrm{Co}}$ changes more with device variations for SOT MTJs than for STT MTJs. This is because when the width of the spin Hall channel changes, $\mathrm{R}_{\text {SHE }}$ changes accordingly. While $\mathrm{J}_{\mathrm{C} 0}$ remains the same, $\mathrm{V}_{\mathrm{C} 0}$ changes with $\mathrm{R}_{\mathrm{SHE}}$ which will drastically affect the SOT switching properties.

There are 3 components that contribute to the total energy consumption in Fig. 7 and Fig. 8. These are the energy for the perturb step, reset step, and logic step. For the perturb and reset steps, the biggest contributing factors for the energy consumption are $\mathrm{J}_{\mathrm{C} 0}$ and switching speed. Having a low $\mathrm{J}_{\mathrm{C} 0}$ along with a low RA product means that $\mathrm{V}_{\mathrm{C} 0}$ is low, therefore, the voltage pulse amplitudes for the perturb and reset steps. However, for the logic step, the resistance of the input MTJs relative to the resistance of the output is another factor that contributes to the energy consumption. If the resistance of the output is low relative to the resistance of the input MTJs, then the amplitude of the voltage pulse needs to increase accordingly.

Figure 9 shows the breakdown of the total energy consumption for AND multiplication for all six MTJ categories. These results show that the breakdown is very similar between research, industrial, and projected STT MTJs, where the reset and logic steps each consume approximately $35-45 \%$ of the total energy consumption and the perturb step consumes around
$15-20 \%$. This means that for STT MTJs, the reduction in total energy consumption almost entirely depends on reducing $\mathrm{J}_{\mathrm{C} 0}$ and the RA product, which is confirmed by the results in Fig. 7. However, there are a few more factors to consider when analyzing the total energy consumption for SOT MTJs. Figure 9 shows that the logic step consumes $95 \%$ of the total energy consumption for industry and research SOT MTJs. On the other hand, the logic step only consumes $44 \%$ of the total energy consumption for projected SOT MTJs. This is because the resistance of the MTJ pillars for the research and industry MTJs are approximately $40 \mathrm{k} \Omega$ to $100 \mathrm{k} \Omega$, which is very high compared to the resistance of the SOT channel, which is around $1.1 \mathrm{k} \Omega$ to $1.3 \mathrm{k} \Omega$. Alternatively, for the projected SOT MTJs, the resistance of the MTJ pillars was around $3 \mathrm{k} \Omega$ in the P -state and $9 \mathrm{k} \Omega$ in the AP-state, which is on the same order of magnitude as the resistance of the SOT channel, which was $\sim 8$ $\mathrm{k} \Omega$. The lower $\mathrm{J}_{\mathrm{C} 0}$ value and faster switching speed for projected SOT MTJs relative to research and industry SOT MTJs certainly are factors in reducing the total energy consumption. However, the results in Fig. 9 show that the biggest factor in reducing the total energy consumption in SOT CRAM is reducing the energy during the logic step. By reducing the RA product of the pillars while maintaining high SOT efficiency in the SOT channel, the total energy consumption can be significantly reduced.

## B. Outlook

Our previous results showed that the energy consumption in SC-CRAM is on the same order of magnitude as that of conventional CRAM [25]. This is quite promising, considering the fact that the energy consumption in conventional CRAM is around 40X lower than for modern near memory processors
[41]. In this study, we demonstrated that the energy consumption in SC-CRAM can be reduced even further by optimizing the intrinsic properties of the MTJ, including reducing $\mathrm{J}_{\mathrm{C} 0}$ and RA product of the pillar. Furthermore, utilizing the SOT switching mechanism in SC-CRAM may reduce the energy consumption even more, especially for large scale applications with large number of NAND operations. Another factor that was not focused on in this study but should be considered is that SC-CRAM benefits from the noise resiliency and robustness to variations associated with stochastic computing, however, in SC-CRAM, there is minimal sacrifice in computation delay and energy for stochastic bit generation since this process is embedded within the computation process.

One of the potential challenges in achieving CRAM cells that utilize the SOT switching mechanism is to create SOT channels with high $\theta_{\mathrm{SH}}$ (low $\mathrm{J}_{\mathrm{C} 0}$ ) along with pillars with low RA products. This is because the SOT materials with the largest $\theta_{\text {SH }}$ values are topological insulators, which typically have large resistivities. This creates potential problems when combining these channels with low RA pillars, since the SOT write current may be shunted into the MTJ pillar rather than contributing to SOT switching. It should be noted that despite this challenge, SOT switching is still preferred over STT switching for two reasons. One is that STT switching has limitations on switching speed and $\mathrm{J}_{\mathrm{C} 0}$. The second is that is less susceptible to breakdown from the write pulse the current direction is adjacent to the free layer rather than across the tunnel junction.

Furthermore, there could be a few solutions to the potential for SOT current shunting. One is that a thin oxide layer could be inserted between the SOT channel and the free layer, which would drastically reduce shunting [56]. However, this would increase the RA product of the pillar, thus increasing the energy consumption during the logic step. Another option is to combine the STT and SOT mechanisms for each step. In previous studies, significant reduction in both switching energy and switching speed has been achieved with this strategy [52]. One method to combine STT and SOT is to use two bias terminals on a three-terminal device. However, this is not an attractive method for the purposes of SC-CRAM since an additional transistor will need to be added to every cell, thus increasing the total circuit area. Another method is to use a two terminal SOT device, which was done in ref. [48]. This strategy allows for the same reduction in switching energy and speed as the three terminal devices, but without adding any transistors to each CRAM cell.

Lastly, another way that the performance of SC-CRAM could be improved is to use new switching mechanisms, e.g. utilizing the voltage controlled magnetic anisotropy (VCMA) [57] and the voltage controlled exchange coupling (VCEC) mechanism and its integration with SOT [58-59]. Previous studies have shown that VCEC switching can be achieved at current densities one order of magnitude lower than for STT switching [59]. Furthermore, our previous studies have shown that VCEC switching can be achieved at current densities as low as $10^{3}$ $\mathrm{A} / \mathrm{cm}^{2}$ when combined with an SOT current [60]. This strategy was not investigated in this study since there are currently no
experimental demonstrations that test the switching speed of the VCEC mechanism. However, SC-CRAM based on VCMA and VCEC switching should be investigated in future studies.

## VI. CONCLUSION

SC-CRAM has the same advantages as any other stochastic computing scheme, which is noise resiliency and low number of logic gates. However, SC-CRAM has additional advantages since stochastic bit-stream generation and computation are performed in parallel. In this study, we analyzed the performance of SC-CRAM for both STT and SOT switching using metrics from data obtained from academic studies, current industrial standards, and projected metrics. Our calculations found that the accuracy of the output is dependent on the TMR ratio of the MTJ pillar for both STT and SOT switching. Additionally, we determined that SC-CRAM cells based on STT switching consume significantly less energy than cells based on SOT switching for MTJs with the current academic and industrial properties. This is primarily due to the large RA product of the MTJ pillars for SOT switching, therefore, large voltages are required to perform logic operations in SOT based SC-CRAM. Based on the projected performance metrics, the energy consumption for SOT CRAM cells can be reduced to levels below STT CRAM cells if the RA product of the MTJ pillar is minimized.

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